

A Ginzburg-Landau Analysis of the Colour Electric Flux Tube*

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In a simulation of SU(2) gauge theory we investigate, after maximal Abelian projection, the dual Maxwell equations for colour field and monopole current distributions around a static quark-antiquark pair $Q\bar{Q}$ in vacuo. Within the dual superconductor picture we carry out a Ginzburg-Landau type analysis of the flux tube profile. As a result we can determine the coherence length of the GL wave function related to the monopole condensate, $\xi = .25(3)$ fm, to be compared to the penetration length, $\lambda = .15(2)$ fm (scaled with the string tension).

1. INTRODUCTION

The QCD vacuum can be viewed as a dual superconductor characterized by a monopole condensate [1]. When embedding a static $Q\bar{Q}$ pair into the vacuum the latter expels by the dual Meissner effect the colour electric fluxlines, thus giving rise to colour confinement. The core of the flux tube is just a normalconducting vortex which is stabilized by solenoidal magnetic supercurrents, \mathbf{k} , in the surrounding vacuum.

These features are to hold in some effective Abelian subsector of the theory. In a series of previous studies of SU(2) and SU(3) gauge theories it has indeed been confirmed that the important d.o.f.'s driving confinement are largely singled out by maximal Abelian gauge (MAG) projection [2], as evidenced e.g. by the string tension [3,4].

In this note we will demonstrate that the validity of this dual superconductivity picture can be extended to colour electric field and monopole current distributions, \mathbf{E} and \mathbf{k} , in rather detail. Refining previous studies along these lines [5–7], we shall perform an unbiased analysis of the Ginzburg-Landau (GL) wave function [8], f , which can be extracted in a sequence of steps from the profile of the flux tube in the transverse center plane (CP) to $Q\bar{Q}$, as seen on the lattice. f carries a second length scale (in addition to the dual photon mass, λ), in form of the coherence length,

ξ , of the condensate; it is the counterpart to the Abelian Higgs mass in the effective GL approach.

2. EVIDENCE FOR DUAL THEORY

It is crucial to start out by establishing the validity of the dual Maxwell theory. We create SU(2) field configurations in the usual way, go through MAG projection into the Abelian world [4] and determine correlators between large Wilson loops and plaquettes therein [9], while \mathbf{k} is defined according to the prescription of our local host [10]. In Fig. 1 we verify that source and sink of \mathbf{E} are strictly localized to the positions of Q and \bar{Q} ! Fig. 2 illustrates that the data on

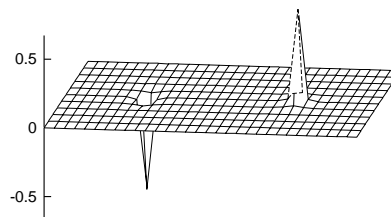


Figure 1. $\text{div } \mathbf{E}$ is strictly localized to the position of Q and \bar{Q} , at separation $r \approx 1.2$ fm.

CP transverse distributions (plotted versus radial distance in lattice units, x , out of core) provides impressive support of the dual Ampère Law.

*Presented by K. Schilling

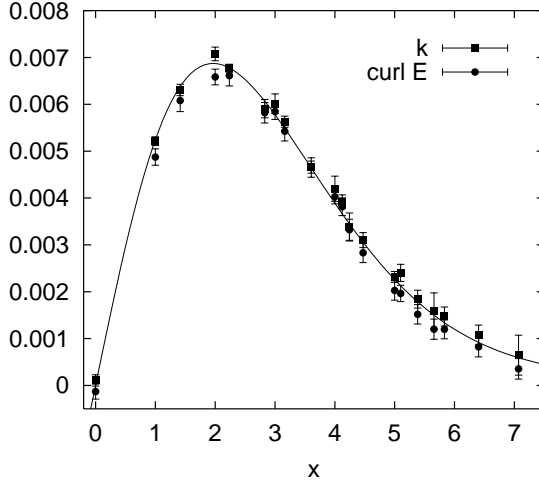


Figure 2. Test of the dual Ampère Law, $\mathbf{k} = \text{curl}\mathbf{E}$.

3. GINZBURG-LANDAU ANALYSIS

Being nonlinear, the GL equations [8] are not at all trivial to solve on a grid:

$$f(x) = f(x)^3 + \xi^2 \left[F(x)^2 - \frac{1}{x} \frac{d}{dx} \left(x \frac{d}{dx} \right) \right] f(x),$$

$$k_\theta(x) = \frac{f(x)^2}{\lambda^2} \frac{\Phi}{2\pi x} F(x), \quad (1)$$

with

$$F(x) = \left(\frac{1}{x} - \frac{2\pi A_\theta(x)}{\Phi} \right). \quad (2)$$

Our approach is to determine f directly from the electric vector potential A_θ (through integration of $\mathbf{E} = \text{curl}\mathbf{A}$) and k_θ via the second GL equation (GLII). For this purpose we employ a parametrization for E_z that respects the boundary conditions of f (BCf) which are rather involved, as they imply the value of $E_z(0)$ [11].

Given the functional form of E_z the dual Ampère Law induces a parametrization for k_θ as well, which at the end leads us to a 4 parameter ansatz to fit both distributions simultaneously. For a discussion of the details we refer to ref. [11]. The two interesting physics parameters resulting from the joint fit to E_z and k_θ , namely the total colour electric flux Φ (expected to have the value *one*) and the penetration length λ come out to be $\Phi = 1.08(2)$, $\lambda = 1.84(8)$, in consistency with the value from a deep London limit

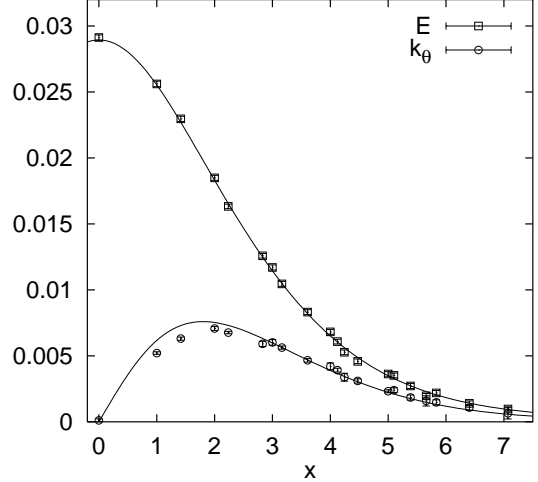


Figure 3. Fitting the azimuthal magnetic supercurrent and longitudinal colour electric field.

analysis, $\lambda = 1.82(7)$ [11]. The simulations are performed on a 32^4 lattice at $\beta = 2.5115$, the lattice spacing being $a \approx 0.081$ fm [9].

The quality of the fit is illustrated in Fig. 3, where we see nice agreement apart from some lattice artifacts in k in the small x regime, $x < 2.2^2$. As a result we would expect to be able to determine f from solving GLII outside this region. The result is shown in Fig. 4 where we plotted the CP transverse distribution of the wave function, together with the profile of the longitudinal field $E_z(x)$. Notice that we attain reasonable statistical accuracy on f , up to about 4.2 lattice spacings out of the vortex core. Beyond this point, we are losing overly in sensitivity for f . Nevertheless one can observe f to approach asymptotia, $f(\infty) = 1$, fairly rapidly.

In order to finally extract the quantity of interest, ξ , it is useful to parametrize f with a functional form that again obeys BCf. To this end we try the usual ansatz $f(x) = \tanh(x/\alpha)$, which is very successful, as evidenced in Fig. 4, with $\alpha = 3.33(5)$, but $\lambda = 1.62(2)^3$. Now that we have a good continuum form of $f(x)$ from GLII at our disposal, we can plug it into GLI and solve the lat-

²We meet non-integer lattice spacings as we use off-axis separations!

³Note that α is not identical to the coherence length as sometimes assumed.

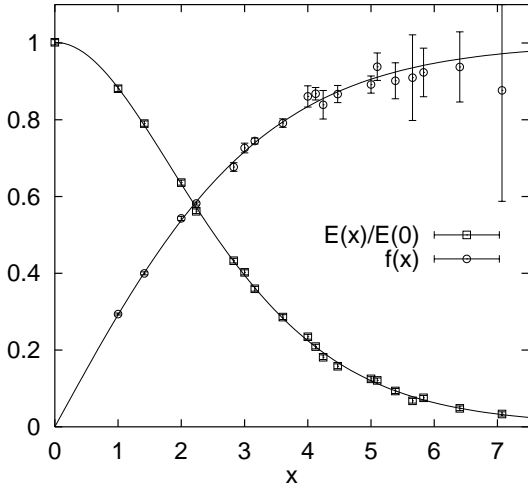


Figure 4. GL wavefunction and colour electric field in transverse profile.

ter in terms of an effective function $\xi(x)$. To the extent that this function proves to be constant we can claim to have arrived at a consistent determination of the coherence length, ξ . In Fig. 5 we exhibit the outcome: indeed ξ shows only a rather mild ($\pm 10\%$) x dependency within our window of observation $2.2 < x < 4.2$. Finally, translating the remaining variation into the systematic error, we can quote the value $\xi = 3.10^{+43}_{-35}$.

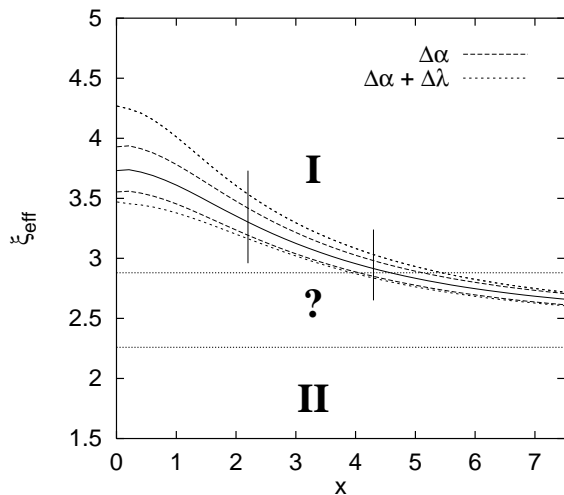


Figure 5. Effective ξ with error bands related to uncertainties in α and λ .

4. DISCUSSION

If one includes the uncertainties in the determination of α and λ , one obtains the error bands for ξ as indicated in Fig. 5. A conservative estimate of the uncertainty in the determination of the penetration length, $\lambda = 1.84^{+20}_{-24}$, blurs the estimated transition point between type I and type II superconductivity, $\kappa = \lambda/\xi = 1/\sqrt{2}$ into an error interval, as indicated in Fig. 5 by the symbol “?”. We conclude that there is *weak evidence* for type I superconductivity.

It would be worthwhile to confirm this tentative conclusion by a scaling analysis and to study interaction among vortices by putting more than one of them into the vacuum.

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